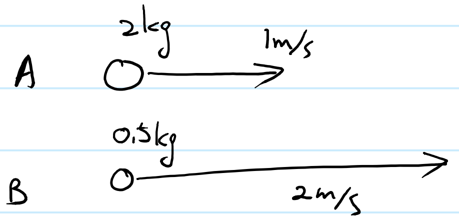
Lecture 6 Impulse and Momentum



applying same force to stop the objects

① Which one travel further in distance before stopping? same

② Which one travel longer in time before stopping? A

To answer the first question①, it is convenient to consider.

$$W = \Delta K$$
$$-F \cdot d = K_f - K_i = -K_i$$

in both case K_i are the same.

To answer ②, we want to relate force and time, and define a quantity similar to the relation between force and space in work-energy theorem.

Define: momentum $\vec{p} = m\vec{v}$

such that: $\Delta \vec{p} = m \Delta \vec{v}$

$$\frac{d\vec{p}}{dt} \leftarrow \frac{\Delta \vec{p}}{\Delta t} = m \frac{\Delta \vec{v}}{\Delta t} \rightarrow m\vec{a} = \vec{F}_{net}.$$

$$\Rightarrow \quad d\vec{p} = \vec{F}_{net} \cdot dt$$

in general. $\longrightarrow$ $$\boxed{\Delta \vec{p} = \vec{p}_f - \vec{p}_i = \int_{t_i}^{t_f} \vec{F}_{net} \, dt}$$

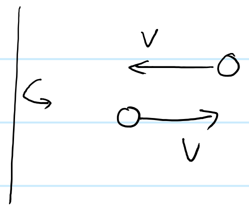
Define: Impulse: $\vec{J} = \int_{t_i}^{t_f} \vec{F}\, dt$ or $\vec{F} \cdot \Delta t$ for constant $\vec{F}$

we have

$$\boxed{\Delta\vec{p} = \vec{J}}$$ Impulse-momentum theorem

c.f. $\Delta K = W$

Ball bouncing



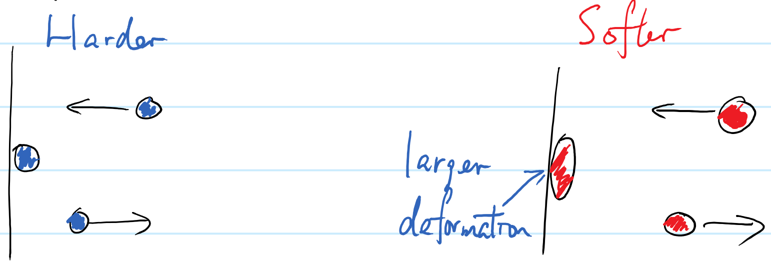
$\vec{p}_i = -mv\hat{i}$ $\quad \Delta\vec{p} = mv\hat{i} - (-mv\hat{i}) = +2mv\hat{i}$

$\vec{p}_f = +mv\hat{i}$

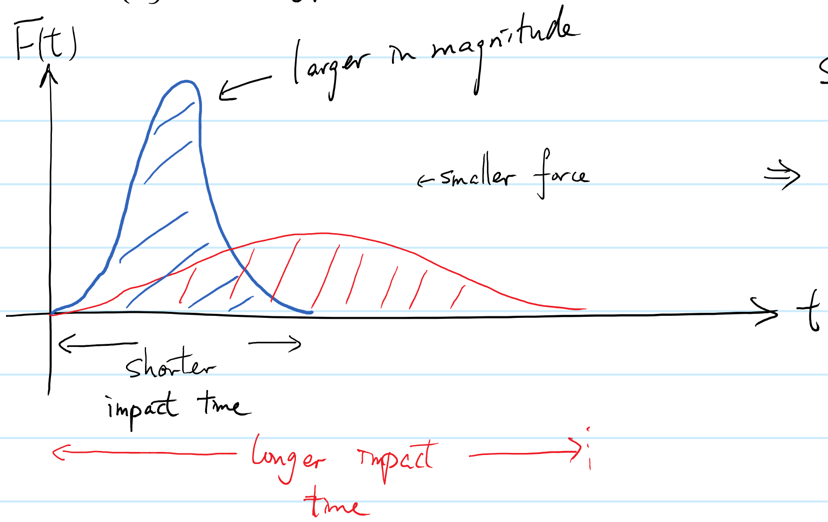
$\Rightarrow \quad \vec{F}\Delta t = 2mv\hat{i}$

But $\vec{F}$ might not be constant in reality: $\vec{F} \to \vec{F}(t)$

for a harder ball, duration of impact is shorter than that of a softer ball



Plot $F(t)$ vs t.



Same $|\vec{J}| = \left|\int F(t)\,dt\right|$

$\Rightarrow$ same area under curve

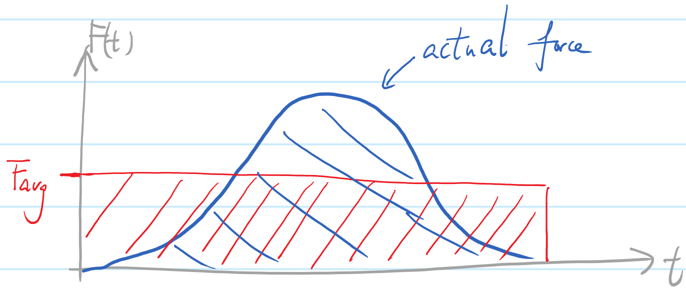
Defining average force, F_{avg}

Average force is defined such that the average force provides the same impulse over the same time. (Δt)

$$\vec{J} = \underbrace{\int_0^{\Delta t} \vec{F}\,dt}_{\text{actual force}} = \underbrace{\vec{F}_{avg}}_{\text{average force}} \cdot \Delta t$$

$$\Rightarrow \quad \vec{F}_{avg} = \frac{1}{\Delta t}\int_0^{\Delta t} \vec{F}\cdot dt = \frac{\vec{J}}{\Delta t}$$

Diagrammatically,



again area under the curves must be the same.

A stiffly built car is not always a safe car in car crash.

Stiff car

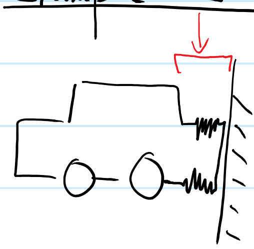
$V \longrightarrow V = 0$

$|\vec{J}| = mv = F \cdot \Delta t$

$$F = \frac{mV}{\Delta t}$$

Δt is small for a stiff car.

Crumple Zone "softer" car

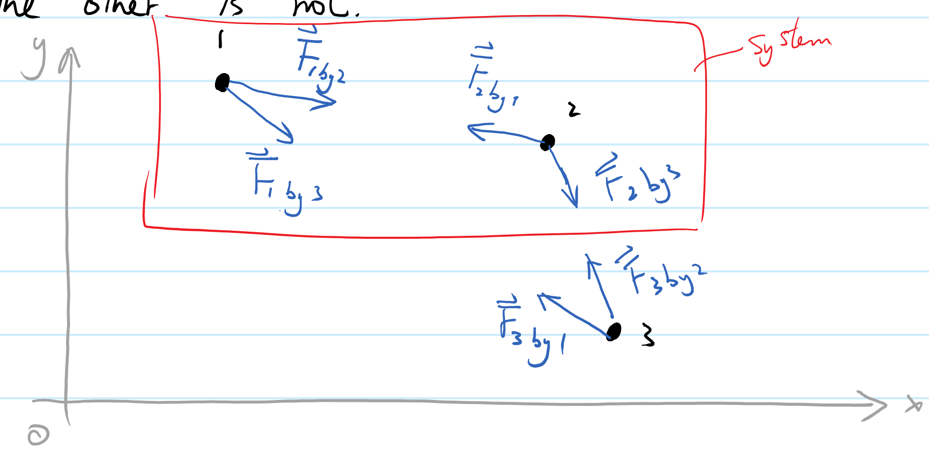


$|\vec{J}| = mV$ But Δt is larger since the head of the car serves as a cushion.

$$\Rightarrow \quad F = \frac{mV}{\Delta t} \downarrow$$

System of particles

Consider three particles, two of them are in the system and the other is not.



Net force acting on the system

$$\vec{F}^{sys}_{net} = \sum_{i=1}^{2} \vec{F}_{net_i} = \vec{F}_{\substack{net\\ on\,1}} + \vec{F}_{\substack{net\\ on\,2}}$$

$$= \vec{F}_{1\,by\,2} + \vec{F}_{1\,by\,3} + \vec{F}_{2\,by\,1} + \vec{F}_{2\,by\,3}$$

$$= \underbrace{\vec{F}_{1\,by\,2} + \vec{F}_{2\,by\,1}}_{\text{internal forces}} + \underbrace{\vec{F}_{1\,by\,3} + \vec{F}_{2\,by\,3}}_{\text{external forces (all by 3)}}$$

$\sum \vec{F}_{internal} = \vec{0}$ ⟵ internal forces

external forces (all by 3)

$\sum \vec{F}^{sys}_{ext}$

$\because \vec{F}_{1\,by\,2} = -\vec{F}_{2\,by\,1}$

Newton's 3rd Law

$$= \sum_i \vec{F}^{sys}_{ext}$$

On the other hand, $\vec{F}^{sys}_{net} = \sum \vec{F}_{net\,i} = \vec{F}_{net,1} + \vec{F}_{net,2} = \dfrac{d\vec{P}_1}{dt} + \dfrac{d\vec{P}_2}{dt}$

$$\Rightarrow \vec{F}^{sys}_{net} = \frac{d}{dt}\vec{P}_{tot} \quad \text{where } \vec{P}_{tot} = \sum_i \vec{P}_i$$

Combining $\vec{F}^{sys}_{net} = \sum \vec{F}_{ext}$ & $\vec{F}^{sys}_{net} = \dfrac{d}{dt}\vec{P}_{tot}$

$$\Rightarrow \sum \vec{F}_{ext} = \frac{d}{dt}\vec{P}_{tot},$$

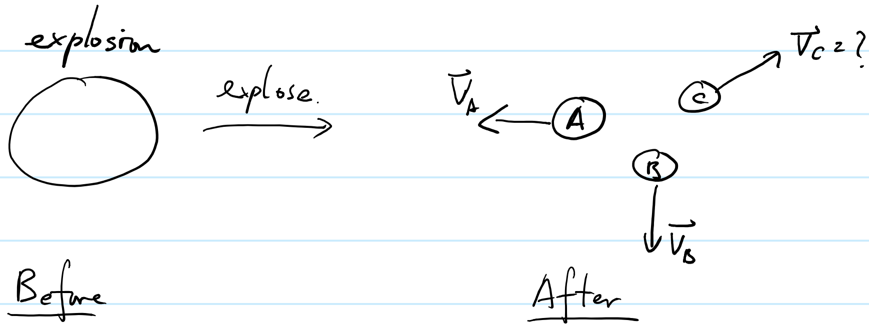
rate of change of total momentum of the sys. equals to sum of external forces only.

If there are no net external forces acting on the system,

$$\Rightarrow \quad \frac{d\vec{P}_{tot}}{dt} = 0 \quad \Rightarrow \quad \Delta\vec{P}_{tot} = \vec{0} \quad \Rightarrow \quad \vec{P}_i = \vec{P}_f$$

<u>Conservation of momentum.</u>

<u>Example</u> explosion



During the explosion, only internal forces are acting on the 3 objects A, B and C.

$$\Rightarrow \quad \vec{P}^{sys}_{before} = \vec{P}^{sys}_{after}$$

$$\vec{P}^{sys}_{before} = (m_A + m_B + m_c) \cdot \vec{0} = \vec{0}$$

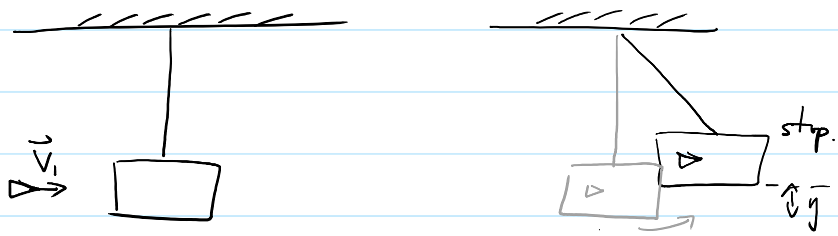
$$\vec{P}^{sys}_{after} = m_A\vec{V}_A + m_B\vec{V}_B + m_c\vec{V}_c$$

$$\Rightarrow \quad \vec{V}_c = -\frac{m_A}{m_c}\vec{V}_A - \frac{m_B}{m_c}\vec{V}_B .$$

Example 1D head-on completely inelastic collision

given m_B, m_W, g

find v_1



$$K_i = \frac{1}{2} m_B v_1^2 \quad \overset{?}{=\!=\!=} \quad U_f = (m_B + m_W)\, g \cdot y$$

using energy conservation?

$$\Rightarrow \quad v_1 = \sqrt{\frac{2(m_B+m_W)}{m_B}\, g y} \quad ? \times \quad \text{WRONG!}$$

When the wood stops the bullet, there exists a friction between the wood and the bullet.

$$\Rightarrow \quad W_{n.c.} \neq 0. \quad \Rightarrow \quad \Delta K + \Delta U \neq 0.$$

Correct analysis:

During collision, $\vec{F}^{sys}_{ext} = \vec{0} \quad \Rightarrow \quad \Delta \vec{p} = 0$ $\quad \because$ There are only friction between the bullet and the wood. That is internal force.

$$\Rightarrow \quad \vec{P}_{\substack{\text{before}\\ \text{collision}}} = \vec{P}_{\substack{\text{after}\\ \text{collision}}}.$$

After the collision, the bullet and the wood move together under tension and gravity.

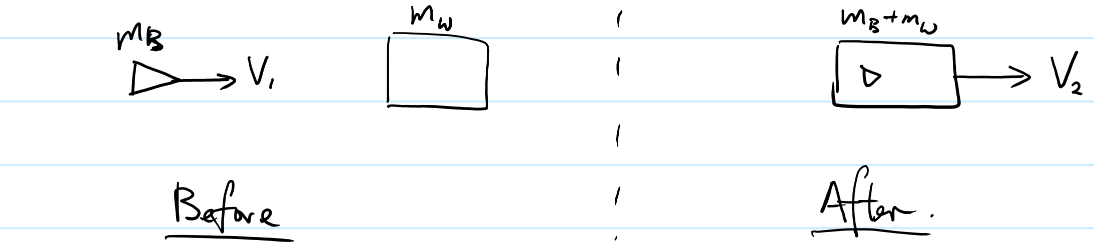
Tension does not do work. $W_T = 0$

Gravity is conservative force,

$$\Rightarrow \quad \Delta K + \Delta U = W_{other} = 0$$

$$\Rightarrow \quad K_i + U_i = K_f + U_f. \quad \text{for the swinging motion.}$$

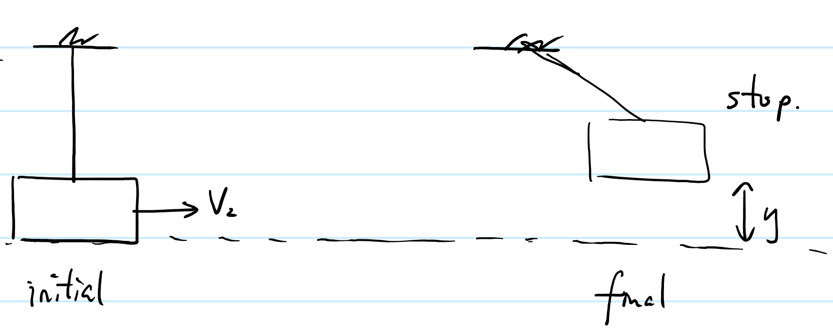
Collision part



$$P_i = m_B V_1 \qquad\qquad P_f = (m_B+m_W)V_2$$

$$P_i = P_f \quad \Rightarrow \quad V_1 = \frac{m_B+m_W}{m_B} V_2$$

Swinging part



$$K_i + U_i = \frac{1}{2}(m_B+m_W)V_2^2 + 0 \quad , \quad K_f + U_f = 0 + (m_B+m_W)g\,y$$

$$K_i + U_i = K_f + U_f$$

$$\Rightarrow \quad \frac{V_2^2}{2} = g\,y$$

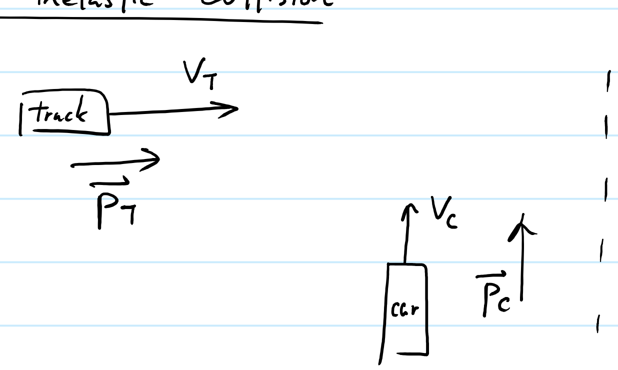
$$\Rightarrow \quad V_2 = \sqrt{2gy}$$

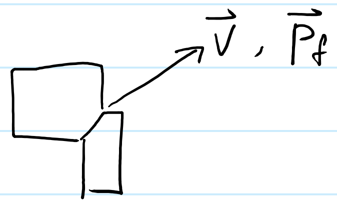
finally we have

$$V_1 = \frac{m_B+m_W}{m_B} V_2 = \frac{m_B+m_W}{m_B}\sqrt{2gy}$$

cf. the wrong answer $v_1 = \sqrt{\frac{m_B+m_W}{m_B} 2gy}$

2D inelastic collision





if momentum is conserved, we have

$$\vec{p}_i = \vec{P}_T + \vec{P}_c = \vec{P}_f$$

During collision,

$$\Delta\vec{p} = \vec{P}_{after} - \vec{P}_{before} = \int_{t_{before}}^{t_{after}} \vec{F}_{ext}\, dt$$

are there any $\vec{F}_{ext}$?

Yes! Before the cars are completely struck and move together, they are already sliding on the road.

$\Rightarrow$ Kinetic friction applies on them.

But to good approximation, the impact time $\Delta t = t_{after} - t_{before}$ is around 0.1 s. The kinetic friction is $f_k = \mu_k (m_c + m_T) g \sim 0.5 \cdot 3000 \cdot 10$ ($\mu_k \sim 0.5$) ~ 15000 N.

$\Rightarrow$ The impulse given by friction during the collision is 1500 Ns.

How is this value compare to the initial momentum of the car and Truck?

Take the car for example, $|\vec{P}_c| = m_c |\vec{V}_c| = 1500$ Ns $\gg$ impulse by friction.

$\Rightarrow$ we can ignore the impulse by the friction and assume the total momentum of the car and truck is conserved.